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I. BIOGRAPHY.

ARTHUR CAYLEY.

BY DR. ALEXANDER MACFARLANE.

ARTHUR CAYLEY was born at Richmond in Surrey, England, August the 16th, 1821. His father, Henry Cayley, was descended from the Cayleys of Brompton, in Yorkshire, but was at the time a merchant of St. Petersburg where he had married a Russian lady. In 1829 his parents took up their permanent residence at Blackheath in England; and Arthur was there educated at a private school for four years. At the age of 14 he was sent to King's College School, London; and the master of that school having observed the promise of a mathematical genius advised the father to educate his son not for his own business, but to enter the University of Cambridge.

In 1838 Arthur Cayley entered Trinity College, Cambridge, at the rather early age of 17. Throughout his undergraduate course he was first at his college examinations by an enormous interval, and he finished his undergraduate career in 1842 by carrying off the two highest honors, namely, the first place, or Senior Wrangler, in the Mathematical Tripos, and the first prize in the competition for the Smith Prizes. Immediately elected a Fellow of his College, he continued to reside at Cambridge for several years, during which time he lectured on mathematics, and also contributed papers to the *Cambridge Mathematical Journal*. His first contribution to that Journal was made, when he was an undergraduate, in 1841.

At that time it was necessary for a Fellow to take Holy Orders, or else resign the fellowship at the end of seven years. Mr. Cayley chose the latter alternative, and became by profession a conveyance in Lincoln's Inn, London. He followed that profession for 14 years with conspicuous ability

and success, and at the same time made many of his most important contributions to mathematical science.

About 1861 the Lucasian professorship of mathematics at Cambridge —the chair made illustrious by Sir Isaac Newton—fell vacant; it was filled by G. G. Stokes, already eminent for his work in mathematical physics, and Senior Wrangler the year before Cayley. However, it was felt desirable to secure Cayley also, and for this purpose the Sadlerian professorship of mathematics was created, which resulted in Cayley marrying and settling down at Cambridge, in 1863.

The duties of the Sadlerian professor were defined as follows: "to explain and teach the principles of pure mathematics, and to apply himself to the advancement of the science". In carrying out the former part of the duties Professsor Cayley did not give the same course of lectures year after year, but each year took for his subject that of the memoir on which he was engaged. As a consequence his students were few, for advanced work of that kind did not pay in the great mathematical examination. How well he carried out the second part of the duties may be inferred from the fact that the Royal Society Catalogue of Scientific Papers enumerates 430 memoirs contributed by him between the years 1863 and 1883, making a total up to the latter date of 724. As he continued active to the last, it is probable that the grand total of his papers does not fall short of 1000. Some of his most celebrated contributions are: Chapters in the Analytical Geometry of (n) Dimensions, On the theory of Determinants, On the theory of linear transformations, Ten Memoirs on Quantics, Memoir on the theory of Matrices, Memoirs on Skew Surfaces, otherwise Scrolls, On the Motion of Rotation of a solid Body, On the triple tangent planes of surfaces of the third order. Several of his achievements are elegantly referred to in a poem written by his colleague Clerk Maxwell in 1874, and addressed to the Committee of subscribers who had charge of the Cayley Portrait Fund:

O wretched race of men, to space confined!
What honor can ye pay to him whose mind
To that which lies beyond hath penetrated?
The symbols he hath formed shall sound his praise.
And lead him on through unimagined ways
To conquests new, in worlds not yet created.

First, ye Determinants, in order row
And massive column ranged, before him go,
To form a phalanx for his safe protection,
Ye powers of the nth root of -1!
Around his head in endless cycles run,
As unembodied spirits of direction.

And you, ye undevelopable scrolls!
Above the host wave your emblazoned rolls,
Ruled for the record of his bright inventions.
Ye cubic surfaces! by threes and nines
Draw round his camp your seven and twenty lines
The seal of Solomon in three dimensions.

March on, symbolic host! with step sublime,
Up to the flaming bounds of Space and Time!
There pause, until by Dickenson depicted,
In two dimensions, we the form may trace
Of him whose soul, too large for vulgar space,
In n dimensions flourished unrestricted.

The portrait was presented to Trinity College, and now adorns their Hall. He is represented as seated at a desk, with quill in hand, and thinking out intently some mathematical idea.

But mathematical science was advanced by Professor Cayley in yet another way. By his immense learning, his impartial judgment, and his friendly sympathy with other workers, he was eminently qualified to act as a referee on mathematical papers contributed to the various societies. Of this kind of work he did a large amount, and of his kindliness to young investigators I can speak from personal experience. Several papers which I read before the Royal Society of Edinburgh were referred to him, and he recommended their publication. Some time after I attended a meeting of the Mathematical Society of London, but the friend who would have introduced me could not be present. Professor Cayley was present, and on finding out who I was, gave me a cordial handshake, and referred in the kindest terms to the papers he had read. His was a cosmopolitan spirit, delighting only in the truth, and friendly to all seekers after the truth.

Among Cayley's papers there are several on a "Question in the Theory of Probabilities". The question was propounded by Boole, and he applied to its solution the general method of "The Laws of Thought". It was afterwards discussed by Wilbraham, Cayley and others in the *Philosophical Magazine*. My attention was drawn to the question when writing the *Principles of the Algebra of Logic*, and I ventured to contribute my idea of the question to the *Educational Times*. On mentioning the matter to Professor Kelland, he intimated pretty plainly that the discussion had been closed by Professor Cayley, and that it was temerity on my part to write anything on the subject. But the great mathematician did not think so; he wrote me a letter discussing the question and my particular way of viewing it, as well as the fundamental ideas in which I differed from Boole.

In 1882 he received a flattering invitation from the trustees of the Johns Hopkins University to deliver a course of lectures on some subject in advanced mathematics. He chose as his subject the Elliptic and Abelian functions; and the impression which his presence created has been well described by Dr. Matz in his brief notice in the January number of the MONTHLY.

Next year he was president of the British Association at the Southport meeting. In his address he spoke of the foundations of mathematics, reviewed the more important theories, traced the connection of pure with applied mathematics, and gave an outline of the vast extent of Modern Mathematics.

He regarded the complex number $a+bi$ as the fundamental quantity of mathematical analysis, and considered that with such a basis, algebra was a complete and bounded science, in which no further imaginary symbols could

spring up. It is the more remarkable that he held such a view, when we consider that early in his career he made a notable contribution to space analysis. Starting from Rodrigues' formulae for the rotation of a solid body, he arrived at the quaternion formula, and was anticipated by Hamilton only by a few months. But Cayley took a Cartesian view of analysis to the last, as is evident from the chapter which he contributed to Tait's *Treatise on Quaternions*. His aim there is to give an analytical theory of quaternions. Hamilton's aim on the other hand was to give a quaternionic theory of analysis. The difference is brought out still more strikingly in a paper printed in the last number of the *Proceedings of the Royal Society of Edinburgh*.

In 1889 the Cambridge University Press commenced the re-publication of his mathematical papers in a collected form. It was calculated that they would occupy 10 quarto volumes; 7 volumes have already appeared; and it is believed that 12 volumes will be required. No mathematician has ever had his works printed in a more handsome manner. In addition he is the author of a separate work on *Elliptic Functions*.

Space fails to enumerate the honors which he received from Universities and Scientific Academies both of the Old and of the New World. But we may mention specally, that from the Royal Society he received a Royal Medal and a Copley Medal; from the Mathematical Society of London the first De-Morgan Medal; and at the instance of the President and Members of the French Academy he was made an Officer of the Legion of Honour.

On the 26th of January he died at Cambridge. His body was laid to rest in Mill Road Cemetery in the presence of official representatives from foreign countries and many of the most illustrious philosophers of England. His spirit still speaks to us from his works, and will continue to speak to many succeeding generations.

II. BIOGRAPHY.

ARTHUR CAYLEY.

BY DR. GEORGE BRUCE HALSTED.

On January 26th, 1895, after long suffering, passed away Professor Cayley, one of the very greatest masters ever known to the world of pure mathematics.

Of the great quaternion of Senior Wranglers of 1840 Leslie Ellis, 1841 Stokes, 1842 Cayley, 1843 Couch Adams, the second alone now remains.